How to Remedy the η-Problem of SUSY GUT Hybrid Inflation via Vector Backreaction

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Abstract.
It is shown that, in supergravity models of inflation where the gauge kinetic function of a gauge field is modulated by the inflaton, we can obtain a new inflationary attractor solution, in which the roll-over of the inflaton suffers additional impedance due to the vector field backreaction. As a result, directions of the scalar potential which, due to strong Kähler corrections, become too steep and curved to normally support slow-roll inflation can now naturally do so. This solves the infamous η-problem of inflation in supergravity and also keeps the spectral index of the curvature perturbation mildly red despite η of order unity. This mechanism is applied to a model of hybrid inflation in supergravity with a generic Kähler potential. The spectral index of the curvature perturbation is found to be 0.97 - 0.98, in excellent agreement with data. The gauge field can act as vector curvaton generating statistical anisotropy in the curvature perturbation. However, this anisotropy could be possibly observable only if the gauge coupling constant is unnaturally small.

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INTRODUCTION

The supersymmetric (SUSY) version [1] of hybrid inflation [2] is undoubtedly one of the most promising types of inflation models that remain into effect today. It naturally arises within SUSY grand unified theory (GUT) models and avoids the massive fine-tuning required in single field inflation models. The slowly rolling inflaton stays sub-Planckian during inflation and, thus, non-renormalizable terms are not totally out of control. Inflation takes place at energies near the SUSY GUT scale in order to yield the observed magnitude of the curvature perturbation. Consequently, the waterfall field can be naturally identified with the GUT Higgs field. SUSY is crucial for inflation since it offers a multitude of flat directions along which inflation can take place. Moreover, the flatness of these directions is not destroyed by radiative corrections as in non-SUSY theories. Actually, in SUSY hybrid inflation, the radiative corrections just provide a gentle logarithmic slope needed for the inflaton to slow-roll.

However, promoting global SUSY to local supergravity (SUGRA), the need of another set of fine-tunings is introduced. Indeed, Kähler corrections to the scalar potential generically give rise [3] to an inflaton mass of order the Hubble parameter H. This is the infamous η-problem of SUGRA inflation, i.e. the fact that the slow-roll parameter η is pushed to order unity by SUGRA corrections:

$$\eta \equiv m_P^2 \frac{V''}{V} \simeq \frac{1}{3}\left(\frac{m}{H}\right)^2 = \mathscr{O}(1) \tag{1}$$

with $m_P = 2.44\times 10^{18}$ GeV being the reduced Planck mass, $m \sim H$ being the inflaton mass, and prime denoting derivative of the scalar potential V with respect to the inflaton field. The scalar spectral index n_s of the curvature perturbation then receives a contribution

$$\delta(n_s - 1) = 2\eta = \mathscr{O}(1), \tag{2}$$

which contradicts its observed approximate scale invariance. Moreover, SUGRA corrections lift the flatness of the inflaton direction and destabilize slow-roll. As a consequence, the number of e-foldings generated is not enough to solve the horizon and flatness problems of standard big bang cosmology. Finally, if the inflaton mass is $m \gtrsim \frac{3}{2}H$, the inflaton cannot generate the necessary density perturbations. Fortunately, hybrid inflation with a minimal Kähler potential is protected from SUGRA corrections by a cancellation and the η-problem does not arise [4]. However, any higher order corrections to the minimal Kähler potential would produce a massive η-problem.

Recently, a surprising solution to the η-problem was proposed [5]. An interaction of the inflaton field with a vector boson field leads to a new inflationary attractor solution, where the vector field backreaction $\mathscr{B}_A$ reduces the effective

inflaton potential slope: $|V'_{\rm eff}| < |V'|$ with $V'_{\rm eff} \equiv V' + \mathscr{B}_A$. This can overcome the η-problem by enabling long-lasting slow-roll inflation to take place even if V is substantially steep and curved. Furthermore, the vector backreaction affects the inflaton equation of motion such that it allows the inflaton to undergo particle production even with $\eta = \mathscr{O}(1)$. We show [6] that the mechanism of vector backreaction also protects the scalar spectral index n_s against excessive contributions from a large η parameter. Applying these results to the standard SUGRA hybrid inflation model with a generic non-minimal Kähler potential, we find [6] that hybrid inflation can be long-lasting and produce a weakly red spectrum of curvature perturbations in agreement with observations.

As expected, the vector field can, in principle, give rise [7, 8] to statistical anisotropy in the curvature perturbation ζ. Note that the observations still allow [9] as much as 30% statistical anisotropy. Moreover, a preferred direction on the microwave sky might be hinted [10] by the unlikely correlation of the low multiples of the cosmic microwave background. One way to generate statistical anisotropy is [11] if the vector field acts as a curvaton [12, 13], i.e. via the vector field perturbations themselves. We apply [6] this to standard SUGRA hybrid inflation to see whether appreciable statistical anisotropy in the spectrum and bispectrum of the curvature perturbation can be generated. We consider natural units, where $c = \hbar = k_B = 1$.

VECTOR SCALING SLOW-ROLL INFLATION

Consider [6] a $U(1)$ gauge symmetry with gauge field B_μ and a complex scalar Higgs field Φ with unit charge. Writing $\Phi = \phi e^{i\varphi}/\sqrt{2}$ and using the gauge invariant combination $h_0 A_\mu \equiv h_0 B_\mu - \partial_\mu \varphi$ (with h_0 being the gauge coupling constant), we obtain the Lagrangian density

$$\mathscr{L} = -\frac{1}{4} f(\sigma) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + \frac{1}{2} h_0^2 \phi^2 A_\mu A^\mu - V_1(\phi), \tag{3}$$

where $F_{\mu\nu}$ is the field strength and $V_1(\phi)$ is the potential for ϕ. The gauge kinetic function f is modulated by the real scalar inflaton field σ:

$$f(\sigma) \equiv \left(\frac{h_0}{h(\sigma)}\right)^2 \tag{4}$$

with $h(\sigma_0) = h_0$ so that $f(\sigma_0) = 1$ and A_μ becomes canonically normalized when σ assumes its vacuum expectation value (VEV) σ_0. The spatial components of the physical vector field acquire a mass $M_A = h_0 \phi_0$ after $U(1)$ breaking (ϕ_0 is the VEV of ϕ).

As inflation homogenizes A_μ, A_0 vanishes [14] and, without loss of generality, we can take $A_\mu = (0,0,0,A_z(t))$. The metric is $ds^2 = dt^2 - a_1^2(t)(dx^2 + dy^2) - a_2^2(t)dz^2$, where $a_{1,2}$ are the scale factors related to the different spatial directions. The average scale factor is $a \equiv (a_1^2 a_2)^{1/3}$ and the average Hubble rate is $H \equiv \dot{a}/a$ (dot denotes derivative with respect to cosmic time). The anisotropic stress induced by the vector field is then

$$\Sigma \equiv \frac{1}{3H}\frac{d}{dt}\ln\left(\frac{a_1}{a_2}\right). \tag{5}$$

The coupling of A_μ to σ through the kinetic function $f(\sigma)$ induces a source term $\mathscr{B}_A \equiv -a_2^{-2} f'(\sigma) \dot{A}_z^2/2$ in the scalar field equation

$$\ddot{\sigma} + 3H\dot{\sigma} + V'(\sigma) + \mathscr{B}_A(\sigma, \dot{A}_z) = 0, \tag{6}$$

where prime denotes derivative with respect to σ, whose potential is $V(\sigma)$. For half of the parameter space and for flat $V(\sigma)$, the system evolves [5] to the standard slow-roll inflationary attractor. On this attractor $\mathscr{B}_A$, Σ, and the vector field energy density ρ_A vanish and, thus, the vector field cannot influence the expansion of the universe. For the other half of the parameter space, $\mathscr{B}_A$ backreacts [5] on the dynamics of σ and the system tends to the vector scaling slow-roll (VSSR) inflationary attractor. On the VSSR attractor, A_μ has a non-negligible effect on the expansion of the universe via its non-vanishing ρ_A and Σ.

On this attractor, $f(\sigma)$ scales [13] as $f_{\rm att} \propto a^{-4}$, which leads to scale-invariant transverse spectra $\mathscr{P}_{L,R}$ of vector field perturbations. From parity conservation, the left and right transverse polarization components $\mathscr{P}_{L,R}$ are identical. If the vector field is massless, its longitudinal component decouples and particle production of this field is highly anisotropic. Thus, the vector field, in this case, can only contribute subdominantly to ζ, but it can still generate substantial statistical anisotropy and anisotropic non-Gaussianity.

The criteria for the existence of the VSSR attractor are [5]

$$\text{Conditions} \quad \begin{cases} \text{I} \quad |\Gamma_f| \gg 1, \\ \text{II} \quad |\Gamma_f| \gg |\lambda_0|, \\ \text{III} \quad \lambda_0 \Gamma_f > 6, \end{cases} \tag{7}$$

where the dimensionless model parameters

$$\Gamma_f(\sigma) \equiv \sqrt{\frac{3}{2}} m_P \left(\frac{f'}{f}\right) \quad \text{and} \quad \lambda_0(\sigma) \equiv \sqrt{\frac{3}{2}} m_P \left(\frac{V'}{V}\right)\Bigg|_{\phi=0} \tag{8}$$

are used and the vector field is considered massless. If the dimensionless model parameters are time-depended, we have additional conditions [5]:

$$A, B, C, D < 1, \tag{9}$$

where

$$\begin{aligned} A &\equiv 4\sqrt{\frac{2}{3}} m_P \left| \frac{\Gamma_f'}{\Gamma_f^2} + \frac{1}{3}\frac{\lambda_0'}{\Gamma_f^2} \right|, \\ B &\equiv 2\sqrt{\frac{2}{3}} m_P \left| \frac{\lambda_0'}{\Gamma_f^2} \right|, \\ C &\equiv 4\sqrt{\frac{2}{3}} m_P \left| \frac{\Gamma_f'}{\Gamma_f^2} + \frac{\lambda_0'}{\Gamma_f^2} - \frac{\lambda_0'\Gamma_f + \lambda_0 \Gamma_f'}{\Gamma_f\left(\lambda_0\Gamma_f - 6\right)} \right|, \\ D &\equiv 4\sqrt{\frac{2}{3}} m_P \left| -\frac{2}{3}\frac{\lambda_0'}{\Gamma_f^2} + \left(\frac{\Gamma_f'}{\Gamma_f^2}\right)\left(\frac{2}{3}\frac{\lambda_0}{\Gamma_f} - \frac{8}{\Gamma_f^2}\right) \right|. \end{aligned} \tag{10}$$

General properties of the VSSR attractor

The vector backreaction $\mathscr{B}_A \equiv -a_2^{-2} f'(\sigma) \dot{A}_z^2/2$ is independent of $\dot{\sigma}$ and thus only modifies the effective slope of the potential $V'_{\rm eff} \equiv V' + \mathscr{B}_A$. Since $f(\sigma) \propto 1/h^2 \to 1$ after the end of inflation, we require that it is always decreasing in time, $\dot{f}(t) < 0$, so that the A_μ remains weakly coupled during inflation. Then, since σ decreases in time during inflation, $f'(\sigma) > 0$ and, thus, $\mathscr{B}_A < 0$ thereby reducing the effective slope of the potential. Once the dimensionless model parameters are slowly varying in time, the backreaction $\mathscr{B}_A$ is proportional to $V'(\sigma)$ and we obtain [5]

$$V'_{\rm eff} \equiv V' + \mathscr{B}_A \simeq \frac{6}{\lambda_0 \Gamma_f} V'. \tag{11}$$

Condition III in (7) then implies that the effective potential slope seen by the inflaton is reduced. So, we can obtain slow-roll inflation with potentials that would normally be too steep for slow-roll. Indeed, on the VSSR attractor, the slow-roll parameters $\varepsilon_{\rm H} \equiv -\dot{H}/H^2$ and $\eta_{\rm H} \equiv -\ddot{H}/2H\dot{H}$ are

$$\varepsilon_{\rm H} \simeq \frac{2\lambda_0}{\Gamma_f} \ll 1 \quad \text{and} \quad \eta_{\rm H} \simeq \frac{2\lambda_0}{\Gamma_f} + \frac{\sqrt{6} m_P}{\Gamma_f}\left(\frac{\lambda_0'}{\lambda_0} - \frac{\Gamma_f'}{\Gamma_f}\right) \tag{12}$$

and slow-roll inflation with $\varepsilon_{\rm H}, \eta_{\rm H} \ll 1$ is possible. From the slow-roll equations

$$3 m_P^2 H^2 \simeq V(\sigma) \quad \text{and} \quad 3H\dot{\sigma} \simeq -V'_{\rm eff}(\sigma), \tag{13}$$

we can find the number of e-foldings

$$N_{\rm att} = \frac{1}{4} \ln \frac{f(\sigma_i)}{f(\sigma_{\rm end})} \tag{14}$$

as the inflaton rolls from σ_i to $\sigma_{\rm end}$, its values at the start and the end of inflation, respectively. The vector-to-scalar field energy density ratio $\mathscr{R}$ does not vanish [5] on the VSSR attractor:

$$\mathscr{R} \equiv \frac{\rho_A}{\rho_\sigma} \simeq \frac{\lambda_0 \Gamma_f - 6}{\Gamma_f^2}, \tag{15}$$

inducing a small anisotropic stress $\Sigma \simeq 2\mathscr{R}/3$ leading [7, 8] to statistical anisotropy in the curvature perturbation ζ.

Curvature perturbation from VSSR inflation

At horizon crossing of the pivot scale $k_* = 0.002\mathrm{Mpc}^{-1}$, the curvature perturbation generated by the inflaton is [6]

$$\frac{2}{5}\zeta_\sigma = \left.\frac{\delta\rho_\sigma}{\rho_\sigma}\right|_* = \left.\frac{1}{5\sqrt{3}\pi}\frac{V^{3/2}}{m_P^3\left|V'_{\rm eff}\right|}\right|_* \simeq \left.\frac{1}{30\sqrt{2}\pi}\frac{V^{1/2}|\Gamma_f|}{m_P^2}\right|_*, \tag{16}$$

where (11) and (13) were used. Considering that the observed $\zeta \simeq 4.8\times 10^{-5}$ (COBE normalization [15]) is generated by the inflaton alone, we have $\zeta_\sigma \simeq \zeta$. The spectrum of the curvature perturbation, in this case, is [6]

$$\mathscr{P}_\zeta \simeq \left.\frac{1}{4\pi^2}\left(\frac{H^2}{\dot\sigma}\right)^2\right|_* \simeq \left.\frac{1}{24\pi^2 m_P^4}\frac{V(\sigma)}{\varepsilon(\sigma)}\left(\frac{\lambda_0\Gamma_f}{6}\right)^2\right|_*, \tag{17}$$

where (13) was used and the slow-roll parameter ε is defined in the usual way $\varepsilon(\sigma) \equiv (m_P^2/2)\,(V'/V)^2$. The spectral index $n_s - 1 \equiv \left.\frac{d\ln\mathscr{P}_\zeta}{d\ln k}\right|_*$ on the VSSR attractor is [6]

$$\begin{aligned} n_s - 1 &\simeq \left(\frac{6}{\lambda_0\Gamma_f}\right)\left[2\eta - 6\varepsilon - 2m_P\sqrt{\frac{2}{3}}\frac{\left(\lambda_0\Gamma_f\right)'}{\Gamma_f}\right] \\ &= -2\left(\frac{6}{\lambda_0\Gamma_f}\right)\left[\varepsilon + m_P\sqrt{\frac{2}{3}}\frac{\lambda_0\Gamma'_f}{\Gamma_f}\right], \end{aligned} \tag{18}$$

where η is the usual slow-roll parameter $\eta(\sigma) \equiv m_P^2\,(V''/V)$. The standard result $n_s - 1 = 2\eta - 6\varepsilon$ is recovered for $V'_{\rm eff} = V'$, i.e. $\lambda_0\Gamma_f = 6$, hence $\left(\lambda_0\Gamma_f\right)' = 0$. We see that n_s is independent of the potential curvature encoded in η and it is easy to obtain a red spectrum favored by the current observational bounds $0.953 \le n_s \le 0.981$ (at 1σ) [15]. The running of the spectral index $n'_s \equiv \frac{dn_s}{d\ln k}$ on the VSSR attractor is also found [6] to be

$$n'_s \simeq 2\varepsilon\left(\frac{6}{\lambda_0\Gamma_f}\right)^2\left\{\eta - 2\varepsilon + 2m_P^2\left[\frac{\Gamma''_f}{\Gamma_f} - 2\left(\frac{\Gamma'_f}{\Gamma_f}\right)^2\right]\right\} \tag{19}$$

and should be compared with the current observational bounds [15] with no gravitational waves: $-0.084 < n'_s < 0.020$ (at 95%cf). The tensor-to-scalar ratio on the VSSR attractor is [6]

$$r \simeq 16\varepsilon\left(\frac{6}{\lambda_0\Gamma_f}\right)^2 = \frac{192}{\Gamma_f^2} \tag{20}$$

with current observational bound [15] with no running of the spectral index $r < 0.36$ (at 95%cf).

Statistical anisotropy and anisotropic non-Gaussianity

Since, during VSSR inflation, the vector field remains massless but with a small non-zero energy density, it could contribute [12, 14] to the primordial curvature perturbation ζ. After the end of inflation, the vector field becomes heavy

with mass M_A and oscillates rapidly behaving like pressureless matter. It can then nearly dominate the energy density and imprint [14] its spectra of perturbations generated during inflation. If $f \propto a^{-4}$, these spectra are [13]

$$\mathscr{P}_{L,R} = \mathscr{P}_\sigma = \left(\frac{H_*}{2\pi}\right)^2 \quad \text{and} \quad \mathscr{P}_\| = 0. \tag{21}$$

The decay rate Γ_A of the oscillating massive vector field is

$$\Gamma_A = \frac{h_0^2 M_A}{8\pi} = \frac{h_0^3 \phi_0}{8\pi} = H_{\rm dec}, \tag{22}$$

where the subscript 'dec' denotes the epoch of vector field decay. For the gravitational effect of the vector field not to be suppressed, its oscillations should last [6] at least one Hubble time before its decay, i.e.

$$\tau \equiv \frac{\Gamma_A}{H_{\rm end}} \simeq \frac{h_0^3 \phi_0 |\Gamma_f(\sigma_*)|}{32\sqrt{6}\pi^2 m_P \zeta} \lesssim 1, \tag{23}$$

where (16) and (22) were used and the subscript 'end' marks the end of inflation. From this, we get

$$h_0 \lesssim \left(\frac{32\sqrt{6}\pi^2 m_P \zeta}{\phi_0 |\Gamma_f(\sigma_*)|}\right)^{1/3}. \tag{24}$$

The anisotropic spectra of perturbations of A_μ generate statistical anisotropy parametrized by g:

$$\mathscr{P}_\zeta(\mathbf{k}) = \mathscr{P}_\zeta^{\rm iso}(k)\left[1 + g\left(\hat{\mathbf{d}}\cdot\hat{\mathbf{k}}\right)^2 + \cdots\right], \tag{25}$$

where $\hat{\mathbf{k}} \equiv \mathbf{k}/k$ and $\hat{\mathbf{d}}$ is the unit vector in the preferred direction. The present upper bound on $|g|$ is 0.3 [9], while the Planck satellite will reduce [16] it to 0.02. Maximal statistical anisotropy is generated [6] if the inflaton decays rapidly after inflation, while the vector curvaton decays later. In this optimal case, the statistical anisotropy is found to be [6]

$$|g| \approx \sqrt{\frac{2}{3}} \frac{\phi_0 \mathscr{R}_{\rm end}}{m_P h_0 \zeta |\Gamma_f(\sigma_*)|}. \tag{26}$$

For a Planck detectable statistical anisotropy $|g| \gtrsim 0.02$ [16], this gives

$$h_0 \lesssim 50\sqrt{\frac{2}{3}} \frac{\phi_0 \mathscr{R}_{\rm end}}{m_P \zeta |\Gamma_f(\sigma_*)|}. \tag{27}$$

The vector curvaton may also generate non-Gaussianity in the curvature perturbation ζ. At present, there is [15] a hint for a non-zero non-linearity parameter $f_{\rm NL}$ characterizing non-Gaussianity: $f_{\rm NL}^{local} = 32 \pm 21$ (at 1σ) . In the optimal case discussed above, we obtain [6]

$$f_{\rm NL} \simeq \frac{5}{3} g^2 \frac{\sqrt{\tau}}{\mathscr{R}_{\rm end}}. \tag{28}$$

For Planck detectable non-Gaussianity $f_{\rm NL} \gtrsim \mathscr{O}(1)$, this leads to

$$h_0 \lesssim \frac{25\sqrt{6}}{3888} \frac{\mathscr{R}_{\rm end}^2}{\pi^2 |\Gamma_f^3(\sigma_*)|} \left(\frac{\phi_0}{m_P \zeta}\right)^5. \tag{29}$$

VECTOR SCALING SUGRA HYBRID INFLATION

We now embed [6] the above vector curvaton into a well motivated model of SUSY GUT hybrid inflation. Consider a simple SUSY GUT model based on the gauge group $G = G_{\rm SM} \times U(1)_{B-L}$ with $G_{\rm SM}$ being the standard model gauge group, which naturally incorporates [1, 4] standard SUSY hybrid inflation. The gauge group G may be thought as part

of a larger GUT gauge symmetry (see e.g. [17]). The model contains a conjugate pair of $G_{\rm SM}$ singlet superfields Φ and $\bar{\Phi}$ with $B-L=+1$ and -1, respectively, which break $U(1)_{B-L}$ by their VEVs and a gauge singlet S triggering the $U(1)_{B-L}$ breaking and acting as our slowly rolling inflaton. Adequate flatness of the inflationary trajectory is guaranteed by a discrete Z_n R-symmetry: $S\to Se^{2\pi i/n}$, $W\to We^{2\pi i/n}$. Note, in passing, that such symmetries arise [18] in many compactified string theories and can effectively act [19] as continuous symmetries.

The most general superpotential relevant for inflation and allowed by the symmetries of the model is

$$W=S\sum_{k_1,k_2=0}^{\infty}A_{k_1k_2}\left(\Phi\bar{\Phi}\right)^{k_1}\left(S^n\right)^{k_2}, \tag{30}$$

where $A_{k_1k_2}$ are coefficients with varying dimensions (for a similar analysis, see [20]). For $n\geq 3$, we rewrite this as

$$W=\kappa S\left(\Phi\bar{\Phi}-M^2\right)+\text{“non-renormalizable terms”}, \tag{31}$$

where $A_{00}=-\kappa M^2$ and $A_{10}=\kappa$ with κ and $M\simeq M_{\rm GUT}$ made positive by field redefinitions ($M_{\rm GUT}$ is the SUSY GUT scale). The non-renormalizable terms are suppressed by powers of m_P.

The scalar potential in SUGRA has the form

$$V=e^{K/m_P^2}\left[F_{\Phi_i}K^{-1}_{ij*}F_{\Phi_j^*}-3\frac{|W|^2}{m_P^2}\right]+\frac{1}{2}\sum_{a,b}\left[{\rm Re}\,f_{ab}(\Phi_i)\right]^{-1}h_ah_bD_aD_b, \tag{32}$$

where K is the Kähler potential, f_{ab} the gauge kinetic functions, and

$$K_{ij*}=\frac{\partial^2K}{\partial\Phi_i\partial\Phi_j^*},\quad F_{\Phi_i}=\frac{\partial W}{\partial\Phi_i}+\frac{W}{m_P^2}\frac{\partial K}{\partial\Phi_i},\quad D_a=\Phi_i(T_a)^i_j\frac{\partial K}{\partial\Phi_j}+\xi_a. \tag{33}$$

Here the subscripts $a,b,\cdots$ label the generators T_a of the gauge group with gauge couplings h_a and ξ_a are Fayet-Iliopoulos D-terms for the $U(1)$ gauge groups. In our model, only the gauge kinetic function f for the $U(1)_{B-L}$ is taken not equal to unity and we have just three fields $\Phi_i=(S,\Phi,\bar{\Phi})$. D-flatness requires that $\bar{\Phi}^*=\Phi e^{i\theta}$, where we choose $\theta=0$ so that the SUSY vacua are contained in this D-flat direction. Then, bringing Φ, $\bar{\Phi}$ on the real axis by a $U(1)_{B-L}$ rotation, we write $\Phi=\bar{\Phi}\equiv\phi/2$, where ϕ is a normalized real scalar field. We also define the normalized real scalar field σ: $|\sigma|\equiv\sqrt{2}|S|$ (see below). The SUSY minimum is at $\sigma=\sigma_0=0$ and $\phi=\phi_0=\pm 2M$.

For $|\sigma|>|\sigma_c|=\sqrt{2}M$, the potential V in global SUSY has a stable flat direction at $\phi=0$ along which inflation can take place driven by the false vacuum energy density κ^2M^4. As $|\sigma|$ crosses its critical value $|\sigma_c|$, the effective mass-squared m_ϕ^2 of ϕ

$$m_\phi^2\simeq\kappa^2\left(|S|^2-M^2\right) \tag{34}$$

becomes tachyonic and inflation ends abruptly by a waterfall.

The Kähler potential K is a real function of the invariants $|S|^2,|\Phi|^2,|\bar{\Phi}|^2,\Phi\bar{\Phi}$, S^n. On the inflationary trajectory, where $\Phi=\bar{\Phi}=0$, the matrix K_{ij*} becomes diagonal and $F_\Phi=F_{\bar{\Phi}}=0$. So the only terms in K which will contribute on this trajectory are

$$K=\sum_{k_1,k_2=0}^{\infty}\frac{|S|^{2k_1}}{m_P^{2k_1+nk_2-2}}\left[a_{k_1k_2}\left(S^n\right)^{k_2}+h.c\right], \tag{35}$$

where $a_{k_1k_2}$ are dimensionless coefficients of order unity. Hence $K=|S|^2-(\alpha/4)|S|^4/m_P^2+\cdots$ with $a_{00}=0$, $a_{10}=1/2$, and $a_{20}=-\alpha/8$, where $|\alpha|\sim 1$ is a real parameter.

The SUGRA scalar potential in (32) restricted on the inflationary trajectory can then be parameterized as

$$V=\kappa^2M^4\sum_{k_1,k_2=0}^{\infty}P_{k_1k_2}\frac{|S|^{2k_1}(S^n)^{k_2}}{m_P^{2k_1+nk_2}}+h.c., \tag{36}$$

where the $P_{k_1k_2}$ are dimensionless coefficients, which are functions of $A_{k_1k_2}$ and $a_{k_1k_2}$. Writing $S=|\sigma|e^{i\vartheta}/\sqrt{2}$, the dimensionless inflationary potential V/κ^2M^4 is found [6] to be

$$\frac{V}{\kappa^2M^4}=1+\frac{\alpha}{2}\left(\frac{\sigma}{m_P}\right)^2+\beta\left(\frac{\sigma}{m_P}\right)^4+2\gamma(n+1)\left(\frac{|\sigma|}{\sqrt{2}m_P}\right)^n\cos n\vartheta+\cdots, \tag{37}$$

where $8\beta = 1+7\alpha/2+2\alpha^2-18(a_{30}+c.c)$, $\gamma = c+a_{01}-a_{11}$ taken real with c from $A_{01} = -c\kappa M^2/m_P^n$. Minimizing V with respect to ϑ, we find that, for $\gamma<0$, $\vartheta = 2\pi k/n$, which by a Z_n transformation can be brought to zero. For $\gamma>0$, V is minimized with $\vartheta = (2k+1)\pi/n$, which by a Z_n transformation can be brought to π/n.

The one-loop radiative corrections to the inflationary potential are given by the Coleman-Weinberg formula [21]:

$$\Delta V_{1-\mathrm{loop}} = \frac{(\kappa M)^4}{32\pi^2}\left(2\ln\frac{\kappa^2\sigma^2}{2Q^2}+f_c(x)\right) \tag{38}$$

with $f_c(x) \equiv (x+1)^2\ln(1+1/x)+(x-1)^2\ln(1-1/x)$, where $x\equiv\sigma^2/2M^2$ and Q is a renormalization scale. They generate a logarithmic slope on the inflationary valley. Note that, for sub-Planckian field values $\sigma<m_P$, the scalar fields remain approximately canonically normalized even with a non-minimal Kähler potential. As we see from (37), for non-canonical Kähler potential K, the inflaton obtains a contribution to its mass-squared $V''(\phi=0)\simeq 3\alpha H^2$ leading to $\eta\simeq\alpha$. Thus, σ would normally be fast-rolling unless α is suppressed, which is the infamous η-problem. In the VSSR attractor, however, we can obtain slow-roll inflation even without fine-tuning the non-canonical Kähler coefficient α.

In our case, the dimensionless model parameter λ_0 in (8) becomes [6]

$$\lambda_0 = \sqrt{\frac{3}{2}}\left[\alpha\left(\frac{\sigma}{m_P}\right)+\left(4\beta-\frac{1}{2}\alpha^2\right)\left(\frac{\sigma}{m_P}\right)^3+\frac{\kappa^2}{8\pi^2}\left(\frac{m_P}{\sigma}\right)+\cdots\right]. \tag{39}$$

We assume here natural values for $|\alpha|\sim 1$ so that the first term in λ_0 dominates over the radiative corrections until the end of inflation. The condition for this is $\kappa<4\pi\sqrt{|\alpha|}\,(M/m_P)$. We will show that a red spectrum can still be obtained, in this case, if the cosmological scales exit during the VSSR attractor. We also find for the first derivative of λ_0 that

$$\sqrt{\frac{2}{3}}m_P\,|\lambda_0'| = |\eta-2\varepsilon| \simeq \left|\alpha+3\left(4\beta-\frac{1}{2}\alpha^2\right)\left(\frac{\sigma}{m_P}\right)^2+\cdots\right|. \tag{40}$$

An exponential gauge kinetic function

The gauge kinetic function, which is holomorphic, cannot contain terms linear in S because of the Z_n R-symmetry, but combinations S^n are allowed. Combinations $\Phi\bar{\Phi}$ are also allowed, but they do not contribute on the inflationary trajectory. We consider [6] an exponential gauge kinetic function

$$f(S^n) = \exp\left[q\left(\frac{S}{M}\right)^n\right] = \exp\left[q\left(\frac{|\sigma|}{\sqrt{2}M}\right)^n e^{in\vartheta}\right], \tag{41}$$

which goes to unity as the inflaton settles into the SUSY vacuum. For $\gamma<0$, where $\vartheta=0$, we choose $q>0$ and for $\gamma>0$, where $\vartheta=\pi/n$, we choose $q<0$ so that the exponent in (41) is always positive:

$$f(\sigma) = e^{|q|\left(\frac{|\sigma|}{\sqrt{2}M}\right)^n}. \tag{42}$$

The dimensionless model parameter $\Gamma_f(\sigma)$ in (8) is then

$$\Gamma_f(\sigma) = |q|n\frac{\sqrt{3}}{2}\left(\frac{m_P}{M}\right)\left(\frac{\sigma}{|\sigma|}\right)\left(\frac{|\sigma|}{\sqrt{2}M}\right)^{n-1}. \tag{43}$$

We see that this parameter is not constant for $n\neq 1$. The VSSR attractor demands that Γ_f and λ_0 have the same sign, so we take the non-canonical Kähler parameter $\alpha>0$. We also choose $\sigma>0$ for definiteness. For $|q|\sim 1$, conditions I and II in (7) for the existence of the VSSR attractor are readily satisfied, while condition III requires that $|q|\alpha n>4$. Since $\lambda_0=\lambda_0(\sigma)$ and $\Gamma_f=\Gamma_f(\sigma)$ are not constants, we have to consider the additional conditions in (9) too. The tightest ones are $A,C<1$, which yield

$$|q| \gtrsim \frac{8}{3}-\frac{8}{3n}, \quad \alpha \gtrsim 4\left[n\left(|q|-\frac{8}{3}\right)\right]^{-1} \tag{44}$$

with the latter being stronger than $|q|\alpha n>4$ required by condition III in (7).

Properties of vector scaling slow-roll inflation

The e-foldings $N_{\rm att}$ during the VSSR attractor from an initial inflaton value σ_i until the end of inflation are found from (14) to be

$$N_{\rm att} = \frac{|q|}{4}\left[\left(\frac{\sigma_i}{\sqrt{2}M}\right)^n - 1\right]. \tag{45}$$

For $\sigma_i < m_P$ and, say, $|q| = n = 3$, we find that $N_{\rm att} \lesssim 10^5$, which is more than enough for solving the horizon and flatness problems. At horizon exit of the pivot scale k_*,

$$\frac{\sigma_*}{\sqrt{2}M} = \left(\frac{4N_*}{|q|} + 1\right)^{1/n}. \tag{46}$$

The reduced effective scalar potential slope is found from (11) to be given by

$$\frac{V'_{\rm eff}}{V'} \simeq \frac{4}{|q|\alpha n}\left(\frac{\sqrt{2}M}{\sigma}\right)^n \ll 1. \tag{47}$$

The slow-roll parameters in (12) become

$$\varepsilon_{\rm H} \simeq \frac{2\alpha}{|q|n}\left(\frac{\sigma}{m_P}\right)^2\left(\frac{\sqrt{2}M}{\sigma}\right)^n, \quad \eta_{\rm H} \simeq \varepsilon_{\rm H} + \frac{2(2-n)}{|q|n}\left(\frac{\sqrt{2}M}{\sigma}\right)^n \tag{48}$$

and remain $\ll 1$ for $n \geq 3$ and $m_P > \sigma > \sigma_c = \sqrt{2}M$. The vector-to-scalar energy density ratio $\mathscr{R}$ increases during VSSR inflation and its value at the end of inflation is

$$\mathscr{R}_{\rm end} \simeq 2\left(\frac{M}{m_P}\right)^2\left[\frac{|q|\alpha n - 4}{(qn)^2}\right]. \tag{49}$$

With $n = |q| = 3$, $\alpha = 4$, we find that $\mathscr{R}_{\rm end} \simeq 1\times 10^{-4}$. For large n, the ratio decreases as $\mathscr{R}_{\rm end} \propto 1/n$.

The curvature perturbation

The primordial curvature perturbation is found [6] from (16) and (46) to be

$$\frac{2}{5}\zeta \simeq \frac{\kappa|q|n}{20\sqrt{6}\pi}\left(\frac{M}{m_P}\right)\left(\frac{4N_*}{|q|} + 1\right)^{(n-1)/n}. \tag{50}$$

The COBE normalization [15] with $M = M_{\rm GUT}$, $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$ then leads to $\kappa \lesssim 1.5\times 10^{-3}$, which readily satisfies the condition for the radiative corrections to be subdominant in (39) with $\alpha = 4$. Note that here the COBE normalization can be satisfied for $M = M_{\rm GUT}$, whereas, in standard SUSY hybrid inflation, M turns out to be [1, 4] somewhat below $M_{\rm GUT}$. For $|q|, \alpha \sim 1$, the scalar spectral index is found from (39), (41), and (46) to be

$$n_s \simeq 1 - \frac{2(n-1)}{nN_*}. \tag{51}$$

So, for $n = 3$, $n_s \simeq 0.978$ and, for $n \gg 1$, $n_s \simeq 0.967$, which fit very well within the 1σ bounds from WMAP [15]. Using (19) and (39), we find that the running of n_s in our model is well approximated by

$$n'_s \simeq -\frac{2(n-1)}{nN_*^2}, \tag{52}$$

which, for $n = 3$, gives $n'_s \simeq -3.6\times 10^{-4}$ and, for $n \gg 1$, $n'_s \simeq -5.4\times 10^{-4}$ well satisfying the WMAP constraints [15]. The tensor-to-scalar ratio is found from (20) and (39) to be

$$r \simeq \frac{256}{(qn)^2}\left(\frac{M}{m_P}\right)^2\left(\frac{4N_*}{|q|} + 1\right)^{2(1-n)/n}, \tag{53}$$

which, for $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, gives $r \lesssim 1\times 10^{-6}$, compatible with WMAP, but probably too small to be observed.

Statistical anisotropy

For the gravitational effect of the vector curvaton not to be suppressed $\tau \lesssim 1$ with τ in (23). This requirement, using (24) and (46), gives

$$h_0^3 \lesssim \frac{32\sqrt{2}\pi^2\zeta}{|q|n}\left(\frac{4N_*}{|q|}+1\right)^{(1-n)/n}, \tag{54}$$

which, for $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, becomes $h_0 \lesssim 0.05$. So, for the SUSY GUT value $h_0 \sim 0.7$ of the gauge coupling constant, the gravitational effect of the vector curvaton is somewhat suppressed. For the statistical anisotropy induced in the spectrum (see (26))

$$|g| \approx \frac{4\sqrt{2}\mathscr{R}_{\rm end}}{3|q|nh_0\zeta}\left(\frac{M}{m_P}\right)^2\left(\frac{4N_*}{|q|}+1\right)^{(1-n)/n}, \tag{55}$$

where $\mathscr{R}_{\rm end}$ is given in (49), to be Planck detectable [16], $h_0 \lesssim 1\times 10^{-4}$ for $\alpha \simeq 4$, $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, which is far smaller than the SUSY GUT h_0. The amplitude of non-Gaussianity (see (28))

$$f_{\rm NL} \simeq \frac{5}{3}g^2\frac{\sqrt{\tau}}{\mathscr{R}_{\rm end}} \tag{56}$$

is Planck detectable, i.e. $f_{\rm NL} \gtrsim \mathscr{O}(1)$, if $h_0 \lesssim 3\times 10^{-10}$ for $\alpha \simeq 4$, $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, which is far too small.

CONCLUSIONS

We showed that, if the inflaton modulates the kinetic function of a vector field, the backreaction to the inflaton's variation allows slow-roll inflation despite the fact that the inflationary potential is too steep as a result of sizable Kähler corrections. Moreover, a mildly red spectral index of inflaton perturbations can be produced eliminating the η-problem of SUGRA inflation.

We have applied the above mechanism to a model of SUGRA hybrid inflation, where the waterfall field is taken to be the Higgs field of a SUSY GUT. The vector field with modulated kinetic function is one of the GUT gauge bosons which become massive at the GUT phase transition. We showed that slow-roll inflation can take place with a generic Kähler potential despite the fact that $\eta = \mathscr{O}(1)$. Moreover, a red spectrum of perturbations is attained, in agreement with observations. Indeed, for an exponential gauge kinetic function, we obtain $n_s \simeq 0.97-0.98$ with negligible running and tensor fraction.

The vector field could contribute to the curvature perturbation ζ if it acts as a vector curvaton. We found that the contribution to statistical anisotropy in ζ is important only when the gauge coupling constant is unnaturally small. This is because the attractor solution is such that the vector field contribution to the energy density is rather small. So, a long period of vector field oscillations is required after the end of inflation for the vector field to become significant. This requires a small decay width of the vector field and thus an unnaturally small gauge coupling constant.

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REFERENCES

1. G. R. Dvali, Q. Shafi, and R. K. Schaefer, *Phys. Rev. Lett.* **73**, 1886–1889 (1994); G. Lazarides, R. K. Schaefer, and Q. Shafi, *Phys. Rev. D* **56**, 1324–1327 (1997).
2. A. D. Linde, *Phys. Lett. B* **249**, 18–26 (1990); *ibid.* **259**, 38–47 (1991); F. C. Adams, and K. Freese, *Phys. Rev. D* **43**, 353–361 (1991); A. D. Linde, *ibid.* **49**, 748–754 (1994).
3. D. H. Lyth, and T. Moroi, *J. High Energy Phys.* **05**, 004 (2004); M. Dine, L. Randall, and S. Thomas, *Phys. Rev. Lett.* **75**, 398–401 (1995); *Nucl. Phys.* **B458**, 291–326 (1996).
4. E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, *Phys. Rev. D* **49**, 6410–6433 (1994) .
5. J. M. Wagstaff, and K. Dimopoulos, *Phys. Rev. D* **83**, 023523 (2011).

6. K. Dimopoulos, G. Lazarides, and J. M. Wagstaff, *JCAP* **02**, 018 (2012).
7. S. Kanno, M. Kimura, J. Soda, and S. Yokoyama, *JCAP* **08**, 034 (2008); M.-a. Watanabe, S. Kanno, and J. Soda, *Phys. Rev. Lett.* **102**, 191302 (2009).
8. B. Himmetoglu, *JCAP* **03**, 023 (2010); C. Pitrou, T. S. Pereira, and J.-P. Uzan, *ibid.* **04**, 004 (2008); A. E. Gumrukcuoglu, B. Himmetoglu, and M. Peloso, *Phys. Rev. D* **81**, 063528 (2010); T. R. Dulaney, and M. I. Gresham, *ibid.* **81**, 103532 (2010); M.-a. Watanabe, S. Kanno, and J. Soda, *Prog. Theor. Phys.* **123**, 1041–1068 (2010); *Mon. Not. Roy. Astron. Soc.* **412**, L83–L87 (2011).
9. N. E. Groeneboom, and H. K. Eriksen, *Astrophys. J.* **690**, 1807–1819 (2009); D. Hanson, and A. Lewis, *Phys. Rev. D* **80**, 063004 (2009); N. E. Groeneboom, L. Ackerman, I. K. Wehus, and H. K. Eriksen, *Astrophys. J.* **722**, 452–459 (2010); Y.-Z. Ma, G. Efstathiou, and A. Challinor, *Phys. Rev. D* **83**, 083005 (2011).
10. K. Land, and J. Magueijo, *Phys. Rev. Lett.* **95**, 071301 (2005); *Mon. Not. Roy. Astron. Soc.* **378**, 153–158 (2007).
11. K. Dimopoulos, M. Karčiauskas, D. H. Lyth, and Y. Rodriguez, *JCAP* **05**, 013 (2009).
12. K. Dimopoulos, *Phys. Rev. D* **76**, 063506 (2007).
13. K. Dimopoulos, M. Karčiauskas, and J. M. Wagstaff, *Phys. Rev. D* **81**, 023522 (2010); *Phys. Lett. B* **683**, 298–301 (2010).
14. K. Dimopoulos, *Phys. Rev. D* **74**, 083502 (2006).
15. E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **192**, 18 (2011).
16. A. R. Pullen, and M. Kamionkowski *Phys. Rev. D* **76**, 103529 (2007).
17. G. Lazarides, I. N. R. Peddie, and A. Vamvasakis, *Phys. Rev. D* **78**, 043518 (2008).
18. N. Ganoulis, G. Lazarides, and Q. Shafi, *Nucl. Phys.* **B323**, 374–392 (1989); G. Lazarides, and Q. Shafi, *ibid.* **B329**, 182–192 (1990).
19. G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, *Phys. Rev. Lett.* **56**, 432–435 (1986).
20. G. Lazarides, and N. Tetradis, *Phys. Rev. D* **58**, 123502 (1998).
21. S. R. Coleman, and E. J. Weinberg, *Phys. Rev. D* **7**, 1888–1910 (1973).